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Discussion

Dynamic Buckling and Post-buckling of Imperfect Cylindrical Shells Under Mechanical and Thermal Loads, Based on the Three-Dimensional Theory of Elasticity[1]

R. L. Citerley.[2] The analysis procedure for any method in applied mechanics requires five steps:

1 Develop the basic governing field equations and their boundary conditions.

2 Examine the possible employment of orthogonal functions to reduce the order of equations into a more manageable set, either by Fourier methods or modal (eigenvalue) methods—this may not be possible.

3 Use a specific numerical approximation for the derivatives of the reduced field equations to a specific numerical accuracy with a known error of approximation, both spatially and temporal. A further approximation may be introduced using finite element methods versus finite difference methods or direct numerical integration methods. In either case, errors are introduced with each method, but the finite element method is known to have more global effects than the local errors associated with the finite difference and integration methods.

4 The application of a solution procedure to the final governing numerical equations. One can apply over-relaxation or matrix inversion solution procedures. Relaxation methods are known to be less accurate with greater potential of error propagation than inversion methods. The latter, however, is much more resource-intensive and time-consuming.

5 Observation of the numerical results and correlate these with a definitive criterion. Being able to differentiate the true behavior of the proposed system of equations including possible nonlinear behavior or the proposed numerical error associated with the solution method becomes the focus.

The authors have made an excellent presentation on the governing equations of a cylindrical composite shell subjected to external loads that can create buckling (step 1). The fact that the authors use three-dimensional elastic theory with a brute force numerical approach raises some specific questions. The questions that the reviewer raises are a result of many years of practical analysis (linear, nonlinear, and dynamic) of composite shells. These questions have not been raised just because of the employment of three-dimensional elasticity, most of which have been presented by others for specific problems of a narrower focus such as vibration or static stress analysis of thick composite shells and not the all-encompassing approach taken by the authors.

The authors claim that the models employed for their examples are in the neighborhood of 150,000 grid points. This means that a banded matrix system of at least the order of 450,000 degrees-of-freedom would result. No mention has been given as to the difference operators used to develop the banded system when imposing specific boundary conditions (step 3). Some special considerations are necessary to eliminate fictitious boundary points in the finite difference grid. Further, the question of nonuniform difference grids needs to be addressed. The system of equations suggested by the authors would be full and not banded if a complete shell of revolution is analyzed. A suggestion has been inferred by the authors that a partial shell was considered by mentioning boundary conditions along a *longitudinal* edge, but no details are given on how the authors would handle nonsymmetric boundary conditions (encountering odd harmonic responses, for example). The reviewer also suspects that the general system of equations would be nonsymmetric, especially if the angle of the layer material composite no longer follows a consistent reference to the principal radii of curvature and/or the bending-membrane parameters of the constitutive equations are full and thus, the shell becomes anisotropic and the governing equations are no longer tridiagonal. With this high order of equations, stability of the solution procedure then comes into question.

For definitions of normal strains and shear strains through the thickness, the endpoints require an adjustment of the difference grid so that no external fictitious grid points would be introduced. Similar arguments are made when introducing the coupling of imperfection geometry (including the rotations about the normal) with the in-plane stress components. For the sake of argument, let us assume that at least five difference equation grid points are needed (and should be more) to adequately represent a smooth function and its derivative. Let us also assume that at least five difference stations are needed to adequately describe a waveform in each of the remaining orthogonal directions.

For many thin composite shells, the buckling waveform in the circumferential direction may be as high as 60. For the axial waveform, its magnitude will be around 20. 150,000 difference stations would be required on this basis. For the axial direction, a minimum of 100 difference stations would be needed to predict adequate bending boundary layer effects. Additional points are required for the zones away from any discontinuities. This raises the difference equations to 750,000 and a full matrix of the same order. The reviewer doubts very seriously that 10^{-12} accuracy could be attained, even with a 64-bit computer word length.

The question of establishing or recognizing a specific buckling mode comes into focus (step 5). Some buckling problems are denoted by a specific growth in a specific or adjacent buckling mode. This mode is usually identified by a specific Fourier harmonic, but can change in value due to coupling (the adding and subtracting of adjacent modes) (step 2). For two of the problems illustrated by the authors, using the procedure developed by Khot (1968) and Khot and Venkayya (1970) and a corresponding numerical procedure provided in the WRC Bulletin 313 (Citerley, 1986), the two axially loaded cases have participating Fourier harmonics that were rather widespread: For the four-ply problem, the critical wave numbers (axial (m), circumferential (n)) are predicted to be (22, 2), (15, 3), (15, 3), and (15, 3) for an

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imperfection geometry of 0.0, 0.5T, T, and 2.0T, respectively. For the three-ply problem, the critical wave numbers were predicted to be (1, 11), (35, 6), (12, 25), and (23, 7) for $h = 2.54$ mm and (2, 3), (7, 2), (5, 3), and (2, 2) for $h = 25.4$ mm. Although the original theory employed to predict these results is based upon Donnell's shell theory, whose limits have been well categorized, one would not expect a major departure from those responses predicted using a three-dimensional theory. Similar results were obtained using the other numerical methods presented in Criterley (1986) and the nonlinear shell programs based on Sander's shell theory using the finite difference (Ball, 1968) and numerical integration (Criterley, 1987) methods. In addition, similar observations were made using the pseudo finite element method program STAGSC-1 (Almroth et al., 1979). Looking only at a w_{max} at a single point may not be easy to identify the buckling mode, especially if there are some 50 to 250 thousand displacements possible and one needs to recognize and identify a buckling pattern that changes with imperfection geometry. The reviewer finds that the reported results have an unusual regularity for the class of problems studied and notes that they differ in behavior from the results obtained from the three aforementioned numerical methods.

Another problem of the paper is that there is an inadequate description of the dynamic analysis methodology. How are the previous displacements of load step and iterations used? Are these quantities considered to be pseudo loads and are accounted for by shifting these to the right-hand side of the system of equations? Are the matrices being inverted and saved or are the system of equations being solved for each time step (step 4)? It is well known that using a Newmark or central difference time operator can produce an unstable solution. Runge-Kutta also falls into this category. Again, is the solution unstable or has the shell reached a bifurcation state? With a nonsymmetric system of equations, the eigenvalue associated with the time step and the matrix equation that represents the equations of motion can be such that a region of instability can be reached—similar to a Mathieu stability phenomenon. The solution could manifest itself with initially a very small error for a considerable length of time. Then, all of a sudden, a large growth in a displacement component is observed. If the solution steps are retraced, the displacement changes are found to exhibit a sign change for each time step. Numerical stability has therefore been encountered and not dynamic instability. One should be cautioned in using only a numerical method using a single observation without some sort of criterion-defining stability, such as applying a variation of energy or a Jacobian determinant. In the reviewer's opinion, this has not been adequately described.

Finally, the title suggest that post-buckling is being investigated. If the brute force method of the numerical solution was used by the authors for all of the classes of analyses presented, no mention has been made as to how the authors circumvent the ill-conditioning of the system of equations just beyond the critical point of bifurcation. Since there are at least two branch paths possible from a bifurcation point (whether static or dynamic), no mention has been made as to how the solution procedure accounts for this possibility.

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Authors' Closure[3]

In the opinion of the authors, investigations of the reviewer are based on a conventional flexural matrix-based finite element analysis stand of view and a previous work by him. Thus, much of his claims are not hold for (does not contribute to) the full three-dimensional theory of elasticity and the proposed solution employed by the authors. He is familiar with a specific solution procedure, and has problems in relating the solution procedure proposed by the authors to what he has in mind.

Due to nonlinearity of the presented governing equations, no explicit stiffness matrix can be introduced unless these equations are linearized. For this reason, instead of using matrix procedures such as the Newton-Raphson method, Broyden-Fletcher-Goldfarb-Shanno method (Bathe, 1996) modal analysis, mode superposition, matrix reduction (Noor and McCom, 1981; Curnier, 1994) or subspace iteration methods, a modified Gauss-Siedel (SOR or successive over-relaxation method that is more rapidly convergent than the GS method) type of solution is used for each iteration within a time step. Thus, a solution vector refreshment is used rather than a matrix procedure. The results are presented for full cylindrical shells with arbitrary axial, radial, and tangential boundary conditions. Due to the variety of the presented examples (examples of static and dynamic buckling under mechanical and thermal loads with symmetric and antisymmetric ply sequences are considered) and due to limitation of the number of the paper pages, additional examples such as a cylindrical shell with asymmetric boundary conditions are not included. Due to the generality of the three-dimensional theory of elasticity, the proposed method is capable of investigating asymmetric and diamond-type buckling. In the presented examples, specifying the load cases was sufficient for understanding the required circumferential and radial boundary conditions. Concepts such as bending-membrane parameters are relevant to the conventional flexural theory that deals with the global behavior of the shell and cannot be defined for the three-dimensional theory of elasticity which is related to the elemental behaviors. On the other hand, it is well known that the prescription of the boundary conditions in the FDM is much simpler than that of FEM. In the paper, FDM is used for replacement of the spatial derivatives. Interlacing or noninterlacing grid points (Brush and Almroth, 1975) can be employed in this procedure and derivative values at the boundary points should be calculated based on the preceding points (Gerald, 1994).

Concepts such as in-plane stresses and rotations about the normals are relevant to global analysis and not to the three-dimensional theory of elasticity. The number of grid points in the axial and circumferential directions in the presented examples are predicted based on the deformed shapes and the number of axial and circumferential wave numbers reported in (Liaw and Yang, 1990; Reddy and Savoia, 1992) and previous works of the authors based on different theories proposed by them (Eslami and Shariyat, 1999; Eslami, Shariyat, and Shakeri, 1998; Shariyat and Eslami, 1999). Generally, grid point distances would be chosen in such a way that no remarkable difference is noticed by decreasing these distances so that effect on the higher modes can be incorporated. Based on this idea, various numbers of grid points are employed where the 150,000 grid points number was a mean value for the different examples. As pointed out earlier, since a vector refreshment scheme rather than a matrix procedure solves the equations, the required computer memory was notably lower. It was mentioned in the paper that the difference in the final results between two numbers of nodes was of the 10^{-12} order. This does not imply that the global error of the numerical procedure itself is of such order.

The authors are aware of interaction of the buckling modes and had confirmed this point in some of their works (Eslami and Shariyat, 1999; Eslami, Shariyat, and Shakeri, 1998; Shariyat and

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