Three-Dimensional Elasticity Solution for the Buckling of Transversely Isotropic Rods: The Euler Load Revisited

C. W. Bert. Professor Kardomeats’ paper is a welcome addition to the literature of the subject. The two Timoshenko formulas cited in the paper by reference to Timoshenko and Gere (1961) are in fact the well-known Engesser formula (Engesser, 1891) and the Haringx formula (Haringx, 1948), referred to by Timoshenko and Gere as the “modified” formula.

For the case of isotropic materials, there has been extensive criticism of the Haringx formula; see Nanni (1971), Ziegler (1982), and Reissner (1982). In particular, in his extensive three-dimensional elasticity analysis, Nanni concluded that the Haringx analysis is less accurate than the Engesser one. This seems to be in direct contradiction to the conclusion of Kardomeats;

an explanation for this apparent discrepancy would be welcomed.

The author has not provided any physical explanations as to why his Eq. (35) is more accurate than either the Engesser or the Haringx prediction. It is conjectured here that the difference is due to the inclusion of axial flexibility in the author’s analysis, while it was tacitly omitted in Engesser’s work and in the particular version of the Haringx work quoted by Timoshenko and Gere (1961).

References


Author’s Closure

The author would like to thank Professor Bert for his gracious comments regarding the paper under discussion. The formulas quoted in the paper as Timoshenko’s (1961) first and second shear correction formulas are indeed the formulas of Engesser (1891) and Haringx (1948) as was pointed out correctly by Professor Bert (Haringx obtained the formula in connection with helical springs and Timoshenko applied Haringx’s approach to bars).

Regarding Nanni’s three-dimensional elasticity approach, a review of his paper (written in German) indicates that Nanni’s (1971) treatment of the problem was based on a stress function approach and an asymptotic scheme with respect to the dimensions of the cross section (e.g., thickness); in this scheme, stresses were expanded in a polynomial series with coefficients that were subsequently determined through compatibility. Therefore, his conclusions (that Engesser’s formula is more accurate than the Haringx analysis) may be influenced by the fact that a certain number of terms were retained (i.e., Nanni’s results can be considered approximations); we may conjecture, therefore, that this conclusion may not hold if more terms in the asymptotic expansion were to be retained.

Instead, our three-dimensional elasticity results for the critical load are derived by numerically solving the determinant of the system of Eqs. (28) (and do not depend on an asymptotic scheme); therefore they can be considered exact. Our conclusion that the Haringx formula is more accurate than Engesser’s was also the feeling of Timoshenko as expressed in Timoshenko and Gere (1961). Nevertheless, it should be noted that in most cases of practical interest, the difference between Engesser or Haringx formulas is small.

Regarding Professor Bert’s conjecture as to why the author’s Eq. (35) is more accurate than either the Engesser or the Haringx formulas, the author would like to thank Professor Bert for offering this reasoning, which seems, indeed, to be a plausible physical explanation.

Author’s Closure

The author would like to thank Professor Chau for drawing his attention to his recent papers (Chau 1993, 1995) on the buckling of short columns. These papers were omitted from the reference list unintentionally since they are too recent and very close to the dates of preparation and submission of the present paper to be found in systematic literature searches at the time the manuscript was authored. Professor Chau’s work, which follows a different method of solution, complements the present paper by including compressible pressure-sensitive material and providing an alternative formulation. The discussion on the roots $s_1$ and $s_2$ is also welcome.

In closing, the author would like to emphasize a few of the main objectives and unique contributions of the present paper such as the derivation of a new, simple, improved column buckling formula, which provides an additional term to the Euler buckling load, and the comparison of the three-dimensional elasticity results with the Timoshenko/Engesser/Haringx transverse shear correction column buckling formulas.

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